O K L A H O M A S T A T E U N I V E R S I T Y

SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING

ECEN 5713 Linear Systems
Spring 2009
Midterm Exam #1

Do All Five Problems

Name : ______________________________

E-Mail Address:_______________________________________

Problem 1:

The impulse response of an ideal lowpass filter is given by

$$g(t) = 2\omega \frac{\sin 2\omega(t - t_0)}{2\omega(t - t_0)}$$

for all *t*, where ω and t_0 are constants. Is the ideal lowpass filter causal? Is it possible to build the filter in the real world?

Problem 2:
Consider a multi-input, multi-output problem involving a satellite of mass *m* in earth orbit specified by its position and velocity in polar coordinates, say

$$\begin{bmatrix} r(\cdot) & \dot{r}(\cdot) & \theta(\cdot) & \dot{\theta}(\cdot) & \phi(\cdot) & \dot{\phi}(\cdot) \end{bmatrix}^T.$$

The input thrusts or forces are written as

$$\begin{bmatrix} u_r(\cdot) & u_\theta(\cdot) & u_\phi(\cdot) \end{bmatrix}^T$$

and maybe applied by using small rocket engines. The equation of motion can then be shown (by using Lagrangian equations) to be

$$\dot{x} = f(x,u) = \begin{bmatrix} \dot{r} \\ r\dot{\theta}^2 \cos^2\phi + r\phi^2 - k/r^2 + u_r/m \\ \dot{\theta} \\ -2\dot{r}\dot{\theta}/r + 2\dot{\theta}\dot{\phi}\sin\phi/\cos\phi + (u_\theta/m)r\cos\phi \\ \dot{\phi} \\ -\dot{\theta}^2\cos\phi\sin\phi - 2\dot{r}\dot{\phi}/r + (u_\phi/m)r \end{bmatrix}.$$

We shall define the output to be the position variables $\{r, \theta, \phi\}$. A free (un-driven) solution of these equation corresponds to the satellite being in a circular equatorial orbit, $x_0(t) = \begin{bmatrix} r_0 & 0 & \omega_0 t & \omega_0 & 0 & 0 \end{bmatrix}^T$, $u_0(t) = 0$, where the radius r_0 and angular velocity ω_0 are such that $r_0^3\omega_0^2 = k =$a constant. However the satellite will deviate from this orbit due to disturbance, and therefore it is of interest to consider the linearized equation about the nominal equatorial orbit. Please exercise the linearization process and find the state space representation.

Problem 3:
Find an *observable* canonical form realization (in minimal order) from SISO continuous-time system given below:

$$5t^2\ddot{y}(t)+(t-1)\dot{y}(t)+e^{-2t}y(t)=2\ddot{u}(t)+2t\dot{u}(t)-t^2u(t).$$

Notice that gain blocks may be *time* dependent. Show the state space representation and its corresponding simulation diagram.

<u>Problem 4</u>:

Find a minimal observable canonical form realization (i.e., its simulation diagram and state space representation) for the following MISO system described by

$$H(s) = \begin{bmatrix} \dfrac{2s+3}{s^3+4s^2+5s+2} & \dfrac{s^4+s^2+2s+2}{s^4+3s^3+3s^2+s} \end{bmatrix}$$

<u>Problem 5</u>:

Show that the two state space representations

$$\dot{x} = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u, \quad y = \begin{bmatrix} 2 & 2 \end{bmatrix} x$$

and

$$\dot{x} = \begin{bmatrix} 2 & 0 \\ -1 & -1 \end{bmatrix} x + \begin{bmatrix} 1 \\ 2 \end{bmatrix} u, \quad y = \begin{bmatrix} 2 & 0 \end{bmatrix} x$$

are realizations of $H(s) = \dfrac{2s+2}{s^2 - s - 2}$. Are they minimal realizations?